$Y \sim p_Y(y)$; $g(Y)$ be a real-valued function of $Y \Rightarrow$ $E[g(Y)] = \sum_y g(y) p_Y(y)$

Example. Let Y be a random variable with probability function given in the table below. Find $E(Y)$, $E(3Y)$, $E(Y^2)$, and $E(3Y - 8)$.

We can find these using the table and the theorem above, by adding a row to the table for each. We'll shortly see another way to find $E(3Y - 8)$.

y	1	2	3	4	Sum
$p_Y(y)$	0.4	0.1	0.2	0.3	1
$y\,p_Y(y)$	0.4	0.2	0.6	1.2	$E(Y) = 2.4$
$3y\,p_Y(y)$	1.2	0.6	1.8	3.6	$E(3Y) = 7.2$
$y^2\,p_Y(y)$	0.4	0.4	1.8	4.8	$E(Y^2) = 7.4$
$(3y - 8)\,p_Y(y)$	– 2.0	– 0.2	0.2	1.2	$E(3Y - 8) = -0.8$

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Definition. The **variance** of a discrete RV, Y, is defined by

$$\sigma^2 = V(Y) = E\left([Y - E(Y)]^2\right) = E\left([Y - \mu]^2\right) = \sum_y (y - \mu)^2\, p(y).$$

The Computing Formula for the Variance.

Since **μ is a constant**, the rules for manipulating summations yield

$$V(Y) = \sum_y (y - \mu)^2\, p(y)$$

$$= \sum_y \left(y^2 - 2\mu y + \mu^2\right) p(y) = \sum_y \left(y^2 p(y) - 2\mu y\, p(y) + \mu^2 p(y)\right)$$

$$= \sum_y y^2 p(y) - 2\mu \sum_y y\, p(y) + \mu^2 \sum_y p(y)$$

$$= E\left(Y^2\right) - 2\mu \cdot E(Y) + \mu^2 \cdot 1$$

$$= E\left(Y^2\right) - 2\mu^2 + \mu^2$$

So,

$V(Y) = E\left(Y^2\right) - \mu^2$ $V(Y) = E\left(Y^2\right) - \left[E(Y)\right]^2$	The Computing Formula for the Variance.

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