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ACSC/MATH 347/447 Final Exam May 11, 2009

Directions.

- **Show all work.** *Full credit will* **not** *be given unless sufficient work is shown or a suitable explanation given.*
- You may **not** use the cdf or pdf functions on a calculator in any problem.
- You may leave binomial coefficients in your answers.
- Part of the credit for each problem will be for style, readability, and mathematical correctness.
- The maximum score is 100 points.
- 2½ hours is allowed for the test.

1. (10 points) If A and B are events with $P(A) = 0.7$, $P(B) = 0.2$, and $P(A \cap B) = 0.1$, find:
   a. $P(A \cup B)$ b. $P(\bar{A})$ c. $P(A \mid B)$
   d. Are A and B independent? *You must explain for credit.*

2. (10 points) Let Y be the number of spots on the face showing after a biased die is rolled. The table below includes part of the probability function, $p(y)$, of the random variable, Y. In each part, in order to receive credit you must write enough to show how you get your answer.
   a. Find $p(6)$. b. Calculate $P(Y \le 3)$. c. Find $E(3Y-2)$.

y	1	2	3	4	5	6	sum
$p(y)$	0.1	0.1	0.2	0.2	0.3		

3. (12 points) Let Y_1 and Y_2 be continuous random variables with joint density function
$$f(y_1, y_2) = \begin{cases} 2y_1 y_2, & 0 \le y_1 \le 2y_2 \le 2 \\ 0, & \text{elsewhere}. \end{cases}$$
   a. Find the marginal probability density function, $f_2(y_2)$, of Y_2.
   b. Find $P(Y_1 \ge Y_2)$. First draw a picture!

4. (18 points) Let Y be a random variable with density function $f(y) = \begin{cases} 2y, & 0 \le y \le 1 \\ 0, & \text{elsewhere.} \end{cases}$
   a. Find the cdf, $F(y)$, of Y. b. Find $E(Y)$.
   c. What is the support of $U = 3Y - 1$? That is, for which values of u is $f_U(u) \ne 0$?
   d. Find the density function of $U = 3Y - 1$.

5. (10 points) At a hospital's emergency room, patients are classified and 20% of them are critical, 30% are serious, and 50% are stable. Of the critical patients, 30% die; of the serious patients, 10% die; and of the stable patients, 1% die.
   a. Find the probability that a randomly selected patient dies.
   b. Given that a patient dies, what is the conditional probability that the patient was classified as critical?

6. (12 points) The random variable Y has a normal distribution with mean 37 and standard deviation 4.
   a. Find $P(30 \le Y \le 40)$.
   b. Find the 67[th] percentile of Y; that is, find the value c so that $P(Y \le c) = 0.67$.

7. (8 points) In a clinical study of a new drug formulated to reduce the effects of rheumatoid arthritis, researchers found that the proportion p of patients who respond favorably to the drug is a random variable that varies from batch to batch of the drug. Assume that p has a probability density function given by

$$f(p) = \begin{cases} 12p^2(1-p), & 0 \le p \le 1 \\ 0, & \text{elsewhere.} \end{cases}$$

Suppose that 20 patients are injected with portions of the drug taken from the same batch. Let Y denote the number showing a favorable response. Find $E(Y)$.

[*Hint*: The statement of the problem gives the distribution of p and the conditional distribution of Y given p. What is $E(Y \mid p)$? How is this related to $E(Y)$?]

8. (12 points) Let Y_1 and Y_2 be random variables such that

$$E(Y_1) = 8, \quad E(Y_2) = 5, \quad V(Y_1) = 9, \quad V(Y_2) = 4, \quad \text{Cov}(Y_1, Y_2) = 5.$$

a. Find $E(Y_1Y_2)$. [*Hint*. Use the computing formula for the covariance.]

b. Find the correlation coefficient of Y_1 and Y_2 .

c. Find the mean and variance of $U = 6Y_1 - 2Y_2$.

9. (8 points) In each part, find $P(Y = 3)$.

a. A bowl contains 50 coins, ten Canadian dimes and 40 US dimes. A coin is selected at random, the type of coin is noted, and then the coin is returned to the bowl. The experiment is repeated 5 times. Let Y denote the number of Canadian dimes selected.

b. A bowl contains 50 coins, ten Canadian dimes and 40 US dimes. A coin is selected at random, the type of coin is noted, and then the coin is set aside. The experiment is repeated 5 times. Let Y denote the number of Canadian dimes selected.

c. A bowl contains 50 coins, ten Canadian dimes and 40 US dimes. A coin is selected at random, the type of coin is noted, and then the coin is returned to the bowl. The experiment is repeated until a Canadian dime is selected. Let Y denote the number of times that the experiment is performed.

d. It is known that 1000 sq. yard bolts of a certain material have an average of 30 flaws per bolt. A customer purchases eight square yards of the material. Let Y be the number of flaws in the material which was purchased. Assume that Y follows a Poisson distribution.

10. (10 points Extra Credit) A random variable Y has an exponential distribution with mean 3. An experimenter is interested in the function $h(Y) = e^{-Y}$.

a. Find $P\left(h(Y) \le \frac{1}{8}\right)$.

b. Find $E\left[h(Y)\right]$.