NAME SOLUTIONS

ACSC/MATH 347/447 Exam #1k 100 Points Total March 2, 2009

Exam 1 Grade:
Course Average:

You must show enough work, or give sufficient explanation, in each problem to clearly indicate how you obtain your answer. **No credit** will be given for a problem if there is insufficient work/explanation.

You may leave binomial coefficients and indicated sums and products in your answers unless otherwise directed in a problem.

1. (8 points) Ms. Jones has four skirts, seven blouses and three sweaters. In how many different ways can she choose two of the skirts, three of the blouses, and one of the sweaters to take on a trip?

Step 1: choose 2 of 4 skirts: $\binom{4}{2}$ ways

Step 2: choose 3 of 7 blouses: $\binom{7}{3}$ ways

Step 3: choose 1 of 3 sweaters: $\binom{3}{1} = 3$ ways

So the number of ways she can choose is

$$\binom{4}{2}\binom{7}{3}\binom{3}{1} = 6 \cdot 35 \cdot 3 = \underline{630}$$

2. (10 points) Suppose that three events, *A*, *B*, and *C*, are defined on a sample space *S*. Use the union, intersection, and complement operations to represent each of the following events symbolically (the first has been answered as a sample):

a. event *A* occurs but event *B* does not occur: $A \cap \bar{B}$

b. at least one of the three events occurs

union

$A \cup B \cup C$

c. all three of the events occur

intersection

$A \cap B \cap C$

d. only event *A* occurs (so not B and not C)

$A \cap \bar{B} \cap \bar{C}$

e. exactly two of the events occur (but it may be any two)

$(A \cap B \cap \bar{C}) \cup (A \cap \bar{B} \cap C) \cup (\bar{A} \cap B \cap C)$

f. none of the three events occurs

$\bar{A} \cap \bar{B} \cap \bar{C} = (\overline{A \cup B \cup C})$

3. (4 points) Color-coded tickets to a museum (with different special exhibit entry times) are to be given to twelve members of a group. There are three red, four green and five blue tickets. In how many different ways can the twelve tickets be distributed to the twelve members of the group if tickets of the same color are identical? Completely evaluate your answer.

Partition the 12 group members into 3 subsets

$$\#\text{ways} = \binom{12}{3\ \ 4\ \ 5} = \frac{12!}{3!\,4!\,5!} = \frac{12\cdot 11\cdot 10\cdot \cancel{9}^{\,3}\cdot \cancel{8}^{\,2}\cdot 7\cdot \cancel{6}\cdot \cancel{5!}}{\cancel{3}\cdot\cancel{2}\cdot 1\cdot \cancel{4}\cdot\cancel{3}\cdot\cancel{2}\cdot\cancel{1}\cdot\cancel{5!}} = \underline{\underline{27{,}720}}$$

or choose 3 people for red, then 4 for green, then 5 for blue

$$\#\text{ways} = \binom{12}{3}\binom{9}{4}\binom{5}{5} = \frac{12!}{3!\,\cancel{9!}}\cdot\frac{\cancel{9!}}{4!\,5!}\cdot 1 = \frac{12!}{3!\,4!\,5!}$$

4. (8 points) A European tour includes four stopovers to be selected from among ten cities. In how many different ways can a person plan such a tour (completely evaluate your answers) :

a. if the order of the stopovers matters;

stop 1   stop2   stop 3   stop4

$$P^{10}_{4} = 10\cdot 9\cdot 8\cdot 7 = \underline{\underline{5040}}$$

b. if the order of the stopovers does not matter?

$$C^{10}_{4} = \binom{10}{4} = \frac{10!}{4!\,6!} = \frac{10\cdot\cancel{9}^{\,3}\cdot\cancel{8}^{\,2}\cdot 7\cdot\cancel{6!}}{\cancel{4}\cdot\cancel{3}\cdot\cancel{2}\cdot 1\cdot\cancel{6!}} = \underline{\underline{210}}$$

5. (12 points) A fair die is tossed seven times and the resulting sequence of seven values is observed.

a. How many sequences are possible?

$$\underline{6}\cdot\underline{6}\cdot\underline{6}\cdot\underline{6}\cdot\underline{6}\cdot\underline{6}\cdot\underline{6} = \underline{6^7}$$

b. How many of these sequences have exactly four ones?

Step 1: Choose 4 blanks for ones: $\binom{7}{4}$ ways

Step 2: Fill the other 3 blanks w/ nonones: 5^3 ways

So there are $\binom{7}{4}\cdot 5^3$ such sequences.

c. How many of these sequences have exactly four odd numbers?

Step 1: Choose 4 blanks for the odds: $\binom{7}{4}$ ways

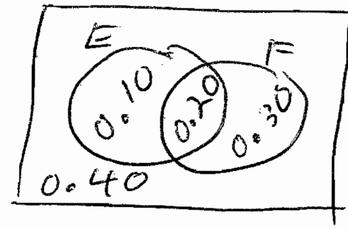
Step 2: Put an odd in each of these: 3^4 ways

Step 3: Put an even in the rest: 3^3 ways

So there are $\binom{7}{4}\cdot 3^4\cdot 3^3$ such sequences

6. (20 points) Let E and F be events with $P(E) = 0.30$, $P(F) = 0.50$ and $P(E \cup F) = 0.60$. Find each of the following, **showing how you get your answer**.

$P(\bar{E}) = 1 - P(E) = 1 - 0.30 = \underline{0.70}$



$P(E \cap F) = P(E) + P(F) - P(E \cup F)$

$= 0.30 + 0.50 - 0.60$

$= \underline{0.20}$

$P(\bar{E} \cap F) = P(E \cup F) - P(E) = 0.60 - 0.30 = \underline{0.30}$

or $= P(F) - P(E \cap F) = 0.50 - 0.20 = 0.30$

$P(\bar{E} \cap \bar{F}) = P(\overline{E \cup F}) = 1 - P(E \cup F)$

$= 1 - 0.60 = \underline{0.40}$

$P(F \mid E) = \dfrac{P(E \cap F)}{P(E)} = \dfrac{0.20}{0.30} = \underline{\dfrac{2}{3}}$

7. (6 points) If A and B are *independent* events with $P(A) = 0.5$ and $P(B) = 0.4$, find $P(A \cup B)$.

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= P(A) + P(B) - P(A)P(B)$ by indep.

$= 0.5 + 0.4 - (0.5)(0.4)$

$= \underline{0.7}$

8. (16 points) Let Y be a random variable with support $\{1, 2, 3, 4, 5\}$ whose probability function $p(y)$ is given by the formula at the right:

$$p(y) = \begin{cases} \dfrac{y}{15}, & y = 1, 2, 3, 4, 5 \\ 0, & \text{elsewhere.} \end{cases}$$

a. Find $P(Y \text{ is odd})$.

$$P(Y \text{ is odd}) = p(1) + p(3) + p(5) = \frac{1}{15} + \frac{3}{15} + \frac{5}{15} = \frac{9}{15}$$

b. Find $P(Y \le 3 \mid Y \text{ is odd})$.

$$P(Y \le 3 \text{ and } Y \text{ is odd}) = p(1) + p(3) = \frac{1}{15} + \frac{3}{15} = \frac{4}{15}$$

$$P(Y \le 3 \mid Y \text{ is odd}) = \frac{P(Y \le 3 \text{ and } Y \text{ is odd})}{P(Y \text{ is odd})} = \frac{4/15}{9/15} = \frac{4}{9}$$

c. Find $E(Y)$ and $V(Y)$.

y	1	2	3	4	5	sum
$p(y)$	$\frac{1}{15}$	$\frac{2}{15}$	$\frac{3}{15}$	$\frac{4}{15}$	$\frac{5}{15}$	1
$y\,p(y)$	$\frac{1}{15}$	$\frac{4}{15}$	$\frac{9}{15}$	$\frac{16}{15}$	$\frac{25}{15}$	$E(Y) = \frac{55}{15} = \frac{11}{3}$
$y^2 p(y)$	$\frac{1}{15}$	$\frac{8}{15}$	$\frac{27}{15}$	$\frac{64}{15}$	$\frac{125}{15}$	$E(Y^2) = \frac{225}{15} = 15$

$$E(Y) = \sum_{y=1}^{5} y\,p(y) = \frac{55}{15} = \frac{11}{3} \text{ from table}$$

$$E(Y^2) = \sum_{y=1}^{5} y^2 p(y) = \frac{225}{15} = 15 \text{ from table}$$

$$V(Y) = E(Y^2) - [E(Y)]^2$$

$$= 15 - \left(\frac{11}{3}\right)^2 = \frac{135 - 121}{9} = \frac{14}{9}$$

9. (6 points) If Y is a random variable with mean 4 and variance 6, find $E(Y^2 - 5Y + 7)$.

$$E(Y^2 - 5Y + 7) = E(Y^2) - 5E(Y) + 7$$

Since $E(Y^2) = V(Y) + [E(Y)]^2 = 6 + 4^2 = 22$,

$$E(Y^2 - 5Y + 7) = 22 - 5 \cdot 4 + 7 = \underline{\underline{9}}$$

10. (10 points) In a certain factory, machines I, II, and III are all producing springs of the same length. Machines I, II, and III produce 1%, 4%, and 2% defective springs, respectively. Of the total production of springs in the factory, Machine I produces 30%, Machine II produces 25%, and Machine III produces 45%.

a. One spring is selected at random from the total springs produced in a given day. Find the probability that the selected spring is defective. [*Hint*: Law of Total Probability]

b. Given that the selected spring is defective , find the conditional probability that it was produced by Machine II.

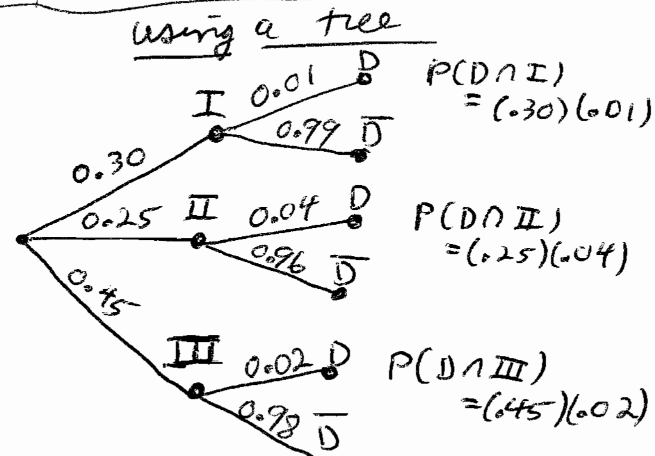
Let I, II, III denote the event that the selected spring was made by the respective machines and let D denote the event the selected spring is defective.

We are told $P(\mathrm{I}) = 0.30$, $P(\mathrm{II}) = 0.25$, $P(\mathrm{III}) = 0.45$

and $P(D|\mathrm{I}) = 0.01$, $P(D|\mathrm{II}) = 0.04$, $P(D|\mathrm{III}) = 0.02$.

(a) By the Law of Total Probability,

$$\begin{aligned} P(D) &= P(\mathrm{I})P(D|\mathrm{I}) + P(\mathrm{II})P(D|\mathrm{II}) + P(\mathrm{III})P(D|\mathrm{III}) \\ &= (0.30)(0.01) + (0.25)(0.04) + (0.45)(0.02) \\ &= 0.0030 + 0.0100 + 0.0090 \\ &= \underline{\underline{0.0220}} \end{aligned}$$



(b) $$\begin{aligned} P(\mathrm{II}|D) &= \frac{P(\mathrm{II} \cap D)}{P(D)} \\ &= \frac{P(\mathrm{II})P(D|\mathrm{II})}{P(D)} \\ &= \frac{0.0100}{0.0220} \\ &= \frac{10}{22} = \underline{\underline{\frac{5}{11}}} \end{aligned}$$