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ACSC/MATH 347/447 Exam #2 100 Points Total November 19, 2007

You must show enough work, or give sufficient explanation, to indicate clearly how you obtain your answer. **No credit** will be given for a problem if there is insufficient work/explanation. **You may not use the statistical functions on a calculator**.

1. (10 points) Let Y be a continuous random variable with density $f(y) = \begin{cases} ce^{-2y}, & 0 < y \\ 0, & \text{elsewhere.} \end{cases}$

   a. Find c so that $f(y)$ is a density function.

   b. Find $P(Y > 2)$.

Clearly indicate how you obtain your answer. **You may not use the statistical functions on a calculator**.

2. (12 points) Measurement errors using a certain measuring device are uniformly distributed on the interval from −5 mm to 5mm.
   a. Find the probability that a measurement made with this device is accurate to within ±3 mm. That is, find the probability that when a measurement is made using this device, the error made lies in the interval from −3 mm to 3 mm.
   b. If 10 items are measured with this device, find the probability that at least 8 are measured accurately to within ±3 mm. Assume that the measurement errors are independent.

Clearly indicate how you obtain your answer. **You may not use the statistical functions on a calculator**.

3. (18 points) The life of a certain type of automobile tire is normally distributed with mean 34,000 miles and standard deviation 4000 miles.
   a. What is the probability that such a tire lasts over 40,000 miles?
   b. What is the probability that such a tire lasts between 30,000 and 35,000 miles?
   c. Given that such a tire has survived 30,000 miles, what is the conditional probability that it survives another 10,000 miles?

Clearly indicate how you obtain your answer. **You may not use the statistical functions on a calculator**.

4. (20 points) Let the random variable Y have **distribution function (cdf)**

$$F(y) = \begin{cases} 0, & y \le 0 \\ \frac{y}{8}, & 0 < y \le 2 \\ \frac{y^2}{16}, & 2 < y \le 4 \\ 1, & 4 < y. \end{cases}$$

a. Find $P(1 < Y \le 3)$.

b. Find the **density function $f(y)$**.

c. Find the mean and variance of Y.

Clearly indicate how you obtain your answer. **You may not use the statistical functions on a calculator**.

5. (8 points) A filling station is supplied with gasoline once a week. If its weekly volume of sales in thousands of gallons is a random variable Y with probability density function

$$f(y) = \begin{cases} 5(1-y)^4, & 0 < y < 1 \\ 0, & \text{otherwise,} \end{cases}$$

what must the capacity of the tank be so that the probability of the supply's being exhausted in a given week is 0.01? That is, find the value c so that $P(Y > c) = 0.01$.

Clearly indicate how you obtain your answer. **You may not use the statistical functions on a calculator**.

6. (12 points) A CD player has a magazine that holds six CDs. The machine is capable of randomly selecting a CD at random and then selecting a song randomly from that CD. Suppose that five CDs are albums by Paul McCartney and one is by Billy Joel. The player selects songs until a song by Billy Joel is played after which the machine is turned off. Give the probability that the machine is turned off after
   a. The sixth song.
   b. At least five songs have been played.
   c. Suppose now that the songs continue to be played until the second song by Billy Joel has been played. Find the probability that at most four songs are played.

Clearly indicate how you obtain your answer. **You may not use the statistical functions on a calculator**.

7. (10 points) An urn has 6 red and 4 white balls. Two balls are chosen at random and without replacement. Let Y be the number of red balls among those selected.
   a. Find the probability function of Y.
   b. Find the moment-generating function of Y.

Clearly indicate how you obtain your answer. **You may not use the statistical functions on a calculator**.

8. (10 points) In the daily production of a certain kind of rope, the number of defects per foot Y is assumed to have a Poisson distribution with mean 2. The profit per foot when the rope is sold is given by X, where $X = 50 - 2Y - Y^2$. Find the expected profit per foot.