NAME SOLUTIONS

ACSC/MATH 347/447 Exam #1j 100 Points Total October 15, 2007

Exam 1 Grade:
Course Average:

You must show enough work, or give sufficient explanation, in each problem to clearly indicate how you obtain your answer. **No credit** will be given for a problem if there is insufficient work/explanation.

You may leave binomial coefficients and indicated sums and products in your answers unless otherwise directed in a problem.

1. (12=3+3+6 points) An art collector owns 10 **different** paintings by famous artists.

   a. In how many different ways can she give one painting to each of her 3 grandchildren?

   $P^{10}_3 = 10 \cdot 9 \cdot 8 = 720$

   b. In how many different ways can she give all 10 paintings to her 3 grandchildren? (There is no restriction other than each of the ten paintings is given to some grandchild.)

   picture $\overline{\#1}$ $\overline{\#2}$ $\cdots$ $\overline{\#10}$

   $\#\text{ways} = 3 \cdot 3 \cdots 3 = 3^{10}$

   c. In how many different ways can she give one painting to each of her 3 grandchildren and donate 4 paintings to the local art museum. (Be careful.)

   step 1: Give one to each grandchild: P^{10}_3 ways

   step 2: Donate 4 to the museum: $\binom{10-3}{4} = \binom{7}{4}$ ways

   so $\#\text{ways} = P^{10}_3 \cdot \binom{7}{4} = 10 \cdot 9 \cdot 8 \cdot \binom{7}{4} = 10 \cdot 9 \cdot 8 \cdot \frac{7 \cdot 6 \cdot 5}{3!}$

   $= (720)(35) = 25200.$

2. (3 points) In how many different ways can one order soup, a sandwich, a dessert, and a drink for lunch, if there is a choice of 4 different soups, 3 kinds of sandwiches, 5 desserts, and 4 drinks?

   Choose soup then sandwich then dessert then drink

   $\#\text{ways} = 4 \cdot 3 \cdot 5 \cdot 4 = 240$

   by the multiplication rule

You must show enough work, or give sufficient explanation, in each problem to clearly indicate how you obtain your answer. **No credit** will be given for a problem if there is insufficient work/explanation.

3. (9=3+4+2 points) A standard deck of cards contains 52 cards, of which 13 are hearts.

   a. How many different 5-card hands can be dealt if the order in which the cards are dealt does not matter?

$$C^{52}_{5} = \binom{52}{5}$$

   b. How many different 5-card hands contain 3 hearts and 2 non-hearts?

choose 3 hearts then choose 2 nonhearts

$$\binom{13}{3} \cdot \binom{39}{2}$$

   c. What is the probability that a randomly selected 5-card hand contains exactly 3 hearts?

$$\frac{\binom{13}{3}\binom{39}{2}}{\binom{52}{5}}$$

4. (5 points) Find the number of distinguishable arrangements of the letters in the word STATISTICS. Completely evaluate your answer.

This is equivalent to a partioning problem.

There are 3 Ss, 3 Ts, 2 Is, 1 A, and 1 C,

so Partition $\{1, 2, 3, \ldots, 10\}$ into subsets of sizes 3, 3, 2, 1, 1 respectively.

$$\#\text{ways} = \binom{10}{3\ 3\ 2\ 1\ 1} = \frac{10!}{3!\,3!\,2!\,1!\,1!} = 50,400$$

<u>alternate solution</u>: Choose 3 positions for S, then 3 positions for T, then 2 for I, then 1 for A, then 1 for C. So

$$\#\text{ways} = \binom{10}{3}\binom{7}{3}\binom{4}{2}\binom{2}{1}\binom{1}{1} = 50,400$$

5. (18 points) Let A and B be events in a sample space, S, with $P(A) = 0.40$, $P(B) = 0.25$ and $P(A \cap B) = 0.10$. Find each of the following, showing how you get your answer (**method** or **formula**). **No credit** will be given unless the method is shown.:

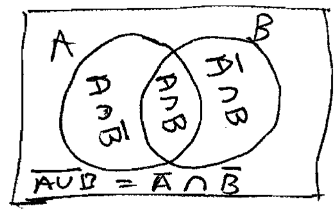
$P(\bar{A}) =$ $1 - P(A) = 1 - 0.40 = \underline{0.60}$

$P(A \cup B) =$ $P(A) + P(B) - P(A \cap B)$

$= 0.40 + 0.25 - 0.10 = \underline{0.55}$

$P(A \cap \bar{B}) =$ $P(A) - P(A \cap B)$

$= 0.40 - 0.10 = \underline{0.30}$



$P(\bar{A} \cap \bar{B}) =$ $P(\overline{A \cup B}) = 1 - P(A \cup B)$

$= 1 - 0.55$

$= 0.45$

$P(B \mid A) =$ $\dfrac{P(A \cap B)}{P(A)} = \dfrac{0.10}{0.40} = 0.25$

Are A and B independent? You must explain your answer.

yes, since $P(B \mid A) = P(B)$.

or yes, since $P(A \cap B) = 0.10 = (0.40)(0.25)$

$= P(A)\,P(B)$

6. (20=3+3+4+3+4+3 points) Let Y be a random variable with support $\{1, 2, 3, 4, 5\}$ whose probability function $p(y)$ is given in the table below. (One value is omitted.)

a. Find $P(Y = 4)$.
b. Find $P(Y \text{ is odd})$.
c. Find $P(Y \leq 2 \mid Y \text{ is odd})$.
d. Find $E(Y)$
e. Find $V(Y)$.
f. Find $E\left(2^Y\right)$

y	1	2	3	4	5	sum
$p(y)$	0.2	0.1	0.1	0.2	0.4	1.0
$y p(y)$	0.2	0.2	0.3	0.8	2.0	$E(Y) = 3.5$
$y^2 p(y)$	0.2	0.4	0.9	3.2	10.0	$E(Y^2) = 14.7$
$2^y p(y)$	0.4	0.4	0.8	3.2	12.8	$E(2^Y) = 17.6$

(a) $P(Y=4) = 1 - p(1) - p(2) - p(3) - p(5) = \underline{0.2}$

(b) $P(Y \text{ is odd}) = p(1) + p(3) + p(5) = 0.2 + 0.1 + 0.4 = 0.7$

(c) $P(Y \leq 2 \mid Y \text{ odd}) = \dfrac{P(Y \leq 2 \text{ \& } Y \text{ odd})}{P(Y \text{ odd})} = \dfrac{P(Y=1)}{P(Y \text{ odd})} = \dfrac{0.2}{0.7} = \dfrac{2}{7}$

(d) $E(Y) = \sum_{y=1}^{5} y p(y) = 3.5$ from row 3 of the table

(e) $V(Y) = E(Y^2) - \{E(Y)\}^2 = 14.7 - (3.5)^2 = 2.45$

from row 3 of the table

(f) $E(2^Y) = \sum_{y=1}^{5} 2^y p(y) = 17.6$

7. (12 points) In a certain population of voters 30% are Republicans, 50% are Democrats and 20% are Independents. It is reported that 40% of the Republicans, 70% of the Democrats and 75% of the Independents favor a certain election issue.

a. Find the probability that a person chosen at random favors the issue in question.

Given: $P(R) = 0.30$

$P(D) = 0.50$

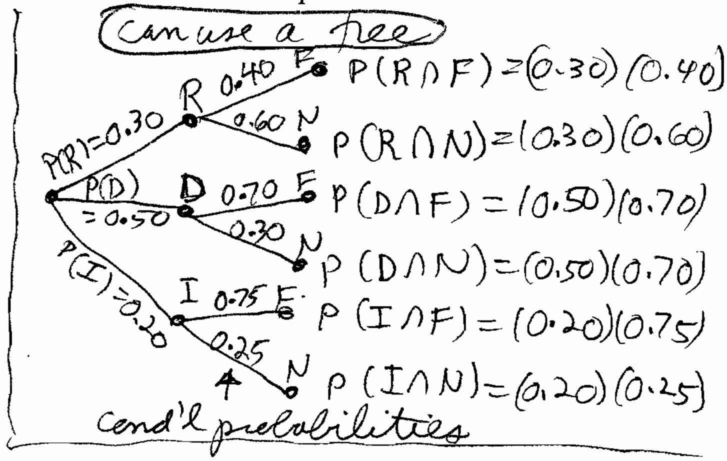
$P(I) = 0.20$

Let F = event person favors issue

$P(F|R) = 0.40$

$P(F|D) = 0.70$

$P(F|I) = 0.75$



so by Law of Total Probability

$$P(F) = P(R)P(F|R) + P(D)P(F|D) + P(I)P(F|I)$$

$$= (0.30)(0.40) + (0.50)(0.70) + (0.20)(0.75)$$

$$= 0.12 + 0.35 + 0.15$$

$$= 0.62$$

b. A person is chosen at random and is found to favor the issue in question. Find the conditional probability that this person is a Democrat. (You may leave your answer as a fraction.)

$$P(D|F) = \frac{P(D \cap F)}{P(F)} = \frac{P(D)P(F|D)}{P(F)}$$

$$= \frac{(0.50)(0.70)}{0.62}$$

$$= \frac{0.35}{0.62}$$

$$= \frac{35}{62}$$

8. (15 points) *You may not use the probability functions on a calculator in this problem.* A multiple-choice test consists of eight questions and three answers to each question (only one of which is correct). A person answers each question by tossing a balanced die and checking the first answer if the top face shows 1 or 2, checking the second answer if it shows 3 or 4, and checking the third answer if it shows 5 or 6. Let the Y denote the number of correct answers the person chooses.

a. Explain why the random variable Y has a binomial distribution and give its parameters, n and p.

Choosing answers by tossing a die makes the $\underline{\underline{n=8}}$ trials independent, and since the die is balanced and 2 of the faces yield the correct answer each toss, the probability of a correct answer each time is $\underline{p = \frac{1}{3}}$. Independent trials and constant p implies a binomial distribution.

b. What is the probability that the person chooses exactly 2 correct answers?

$$P(Y=2) = \binom{8}{2}\left(\frac{1}{3}\right)^2\left(\frac{2}{3}\right)^6$$

c. What is the probability that the person chooses at least one correct answer? Completely evaluate this answer. (Part of your credit will be for the efficiency of your method.)

$$P(Y \geq 1) = 1 - P(Y < 1) = 1 - P(Y=0)$$
$$= 1 - \binom{8}{0}\left(\frac{1}{3}\right)^0\left(\frac{2}{3}\right)^8 = 1 - \left(\frac{2}{3}\right)^8 = 0.961$$

d. What is the expected number of correct answers the person chooses?

$$E(Y) = np = 8\left(\frac{1}{3}\right) = \frac{8}{3}$$

e. What is the variance of Y, the number of correct answers the person chooses?

$$V(Y) = npq = 8\left(\frac{1}{3}\right)\left(\frac{2}{3}\right) = \frac{16}{9}$$

or $V(Y) = np(1-p)$

9. (6 points) If Y is a random variable such that $E(Y) = -3$ and $V(Y) = 7$, find:

a. $E(5Y + 4)$ $= 5E(Y) + 4$

$= (5)(-3) + 4$

$= -11$

b. $V(5Y + 4)$ $= 5^2 V(Y)$

$= (25)(7)$

$= 175$

c. $E(Y^2)$ $= V(Y) + [E(Y)]^2$ from the computing Formula for the Variance

$= 7 + (-3)^2$

$= 16$