MATH 347-24 Quiz 8g Solutions (20 Points Total) November 8, 2004

1. Let Y be a continuous random variable with cumulative distribution function (cdf) given

$$\text{by } F(y) = \begin{cases} 0, & y \le 0 \\ \dfrac{y^2}{8}, & 0 \le y < 2 \\ \dfrac{y}{4}, & 2 \le y < 4 \\ 1 & y \ge 4 \end{cases}$$

a. (9 points) Use $F(y)$ to evaluate the following probabilities:

(i) $P(Y \le 1) = F(1) = \dfrac{1^2}{8} = \dfrac{1}{8}$

(ii) $P(1 \le Y \le 3) = F(3) - F(1) = \dfrac{3}{4} - \dfrac{1}{8} = \dfrac{5}{8}$

(iii) $P(Y > 2.4) = 1 - F(2.4) = 1 - \dfrac{2.4}{4} = 0.4$

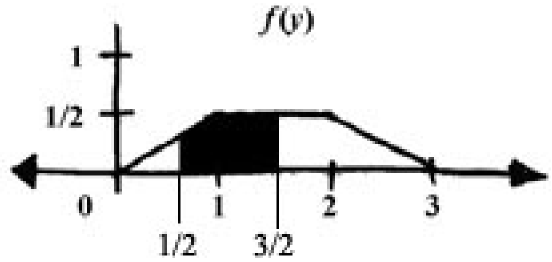
b. (4 points) Find the density function (pdf), $f(y)$, of Y.

$$f(y) = F'(y) = \begin{cases} \dfrac{d}{dy}(0) = 0, & y \le 0 \\ \dfrac{d}{dy}\left(\dfrac{y^2}{8}\right) = \dfrac{y}{4}, & 0 < y < 2 \\ \dfrac{d}{dy}\left(\dfrac{y}{4}\right) = \dfrac{1}{4}, & 2 < y < 4 \\ \dfrac{d}{dy}(1) = 0 & y > 4 \end{cases}$$

Note: The derivative of $F(y)$ is not defined for $y = 0, 2,$ or 4. So we can either say that $f(y)$ is not defined at these three places, or we can define it to be any convenient value at these three places.

2. Let Y be a continuous random variable whose density function, $f(y)$, and the graph of $f(y)$ are given below.

$$f(y) = \begin{cases} \dfrac{y}{2}, & 0 < y \le 1 \\ \dfrac{1}{2}, & 1 < y \le 2 \\ \dfrac{3-y}{2}, & 2 < y \le 3 \\ 0, & \text{elsewhere} \end{cases}$$



a. (3 points) Shade the region in the graph of the density function whose area is $P\left(\frac{1}{2} \le Y \le \frac{3}{2}\right)$.

b. (4 points) Evaluate $E(Y)$.

$$E(Y) = \int_{-\infty}^{\infty} y\, f(y)\, dy$$

$$= \int_{-\infty}^{0} y \cdot 0\, dy + \int_{0}^{1} y \cdot \frac{y}{2}\, dy + \int_{1}^{2} y \cdot \frac{1}{2}\, dy + \int_{2}^{3} y \cdot \frac{3-y}{2}\, dy + \int_{3}^{\infty} y \cdot 0\, dy$$

$$= 0 + \int_{0}^{1} \frac{y^2}{2}\, dy + \int_{1}^{2} \frac{y}{2}\, dy + \int_{2}^{3} \left[\frac{3y}{2} - \frac{y^2}{2}\right] dy + 0$$

$$= \left[\frac{y^3}{6}\right]_0^1 + \left[\frac{y^2}{4}\right]_1^2 + \left[\frac{3y^2}{4} - \frac{y^3}{6}\right]_2^3$$

$$= \left[\frac{1}{6} - \frac{0}{6}\right] + \left[\frac{4}{4} - \frac{1}{4}\right] + \left[\left(\frac{27}{4} - \frac{27}{6}\right) - \left(\frac{12}{4} - \frac{8}{6}\right)\right]$$

$$= \frac{1}{6} + \frac{3}{4} + \frac{15}{4} - \frac{19}{6}$$

$$= \frac{3}{2}$$