

Show all your work.

60 points maximum score

50 minutes allowed

You do not need to evaluate binomial coefficients.

The test is printed on both sides of the paper.

1. (6 points) A sample space consists of five simple events (outcomes), E_1 , E_2 , E_3 , E_4 , and E_5 , which are mutually exclusive. In each part tell whether the assignment of probabilities is permissible and **explain your answers**:

a. $P(E_1) = 0.21$, $P(E_2) = 0.20$, $P(E_3) = 0.50$, $P(E_4) = 0.01$, and $P(E_5) = 0.06$.

b. $P(E_1) = 0.18$, $P(E_2) = 0.20$, $P(E_3) = 0.25$, $P(E_4) = 0.15$, and $P(E_5) = 0.22$.

c. $P(E_1) = 0.10$, $P(E_2) = 0.30$, $P(E_3) = 0.10$, $P(E_4) = 0.60$, and $P(E_5) = -0.10$.

2. (18 points) If A and B are events with $P(A) = 0.7$, $P(B) = 0.2$, and $P(A \cap B) = 0.1$, find:

a. $P(A \cup B)$

b. $P(\bar{A})$

c. $P(A \cap \bar{B})$

d. $P(A|B)$

e. Are A and B independent? *You must explain for credit.*

3. (10 points) A club contains 10 men and 20 women. Five members are to be chosen to form an executive committee.
- In how many different ways can the committee be chosen?

- If the members of the committee are chosen at random, find the probability that the committee has exactly two men.

4. (10 points) A balanced die is tossed five times and the number on the uppermost face is recorded each time. Let the sample space, S , be the set of all 5-tuples which are possible, and let A be the event that the five numbers recorded are distinct. Find $P(A)$.

5. (8 points) The completion of a construction job may be delayed because of a strike. The probabilities are 0.60 that there will be a strike, 0.85 that the construction job will be completed on time if there is no strike, and 0.35 that the construction job will be completed on time if there is a strike. Let C be the event that the job is completed on time and let K be the event that there is a strike.
- Use the *Law of Total Probability* to compute $P(C)$, the probability that the construction job was completed on time.
 - Given that the job was completed on time, find the probability that there was a strike.

6. (8 points) A fair coin is tossed repeatedly until *either* a head appears *or* the coin has been tossed four times. Thus, for example, if a head appears on the first toss, the coin is only tossed once and if a head never appears, the coin is tossed four times. Let an outcome (simple event) be the sequence of heads and tails observed.

- a. List the elements in the sample space, S , and assign reasonable probabilities to each.
- b. Let the random variable, Y , be the number of times the coin is tossed.
 - (i) What is the support of Y ?
 - (ii) Table the values of the probability function, $p(y)$, of the random variable, Y .