Problems on Mixed Distributions

1. Problem 4.156 on page 213 of WMS.
2. Problem 4.157 on page 213 of WMS.
3. Let X be a random variable having distribution function

$$F(x) = \begin{cases} 0, & x < 0 \\ \dfrac{x^2}{4}, & 0 \le x < 1 \\ \dfrac{x+1}{4}, & 1 \le x < 2 \\ 1, & 2 \le x. \end{cases}$$

   a. Carefully sketch the graph of $F(x)$.
   b. Find the mean and the variance of X.
   c. Find $P\left(\frac{1}{4} < X < 1\right)$, $P\left(\frac{1}{4} < X \le 1\right)$, $P\left(X = \frac{1}{2}\right)$, $P(X = 1)$, $P\left(\frac{1}{2} \le X < 2\right)$, and $P\left(\frac{1}{2} \le X \le 2\right)$.

4. The weekly gravel demand X (in tons) has density function

$$f(x) = \begin{cases} 0. & x \le 0 \\ \left(\dfrac{1}{5}\right) e^{-x/5}, & 0 < x < \infty. \end{cases}$$

However, the owner of the gravel pit can only produce at most 4 tons of gravel per week. Compute the expected value of the tons sold per week by the owner.

***4.156** The duration Y of long-distance telephone calls (in minutes) monitored by a station is a random variable with the properties that

$$P(Y = 3) = .2 \quad \text{and} \quad P(Y = 6) = .1.$$

Otherwise, Y has a continuous density function given by

$$f(y) = \begin{cases} (1/4) y e^{-y/2}, & y > 0, \\ 0, & \text{elsewhere.} \end{cases}$$

The discrete points at 3 and 6 are due to the fact that the length of the call is announced to the caller in three-minute intervals and the caller must pay for three minutes even if he talks less than three minutes. Find the expected duration of a randomly selected long-distance call.

***4.157** The life length Y of a component used in a complex electronic system is known to have an exponential density with a mean of 100 hours. The component is replaced at failure or at age 200 hours, whichever comes first.

a Find the distribution function for X, the length of time the component is in use.

b Find $E(X)$.