NAME SOLUTIONS

A380/M480 Quiz 2a February 8, 2005 20 pts. max Show all work; no work $\Rightarrow$ no credit.

1. Let $P(A \cap B) = 0.2$, $P(A) = 0.6$, and $P(B) = 0.5$. Then $P(A' \cup B') =$

A. 0.1 B. 0.3 C. 0.7 D. 0.8 E. 0.9

$$P(A' \cup B') = 1 - P((A' \cup B')')$$
$$= 1 - P(A \cap B)$$
$$= 1 - 0.2 = 0.8$$

2. Let X_1 and X_2 be independent random variables, each with density function $f(x) = \begin{cases} 2-2x & \text{for } 0 < x < 1 \\ 0, & \text{otherwise} \end{cases}$.

What is the probability that exactly one of the two variables exceeds $\frac{1}{2}$?

A. $\frac{1}{16}$ B. $\frac{6}{16}$ C. $\frac{7}{16}$ D. $\frac{8}{16}$ E. $\frac{9}{16}$

Let E = event $X_1 > \frac{1}{2}$, F = event $X_2 > \frac{1}{2}$. Then E, F are indep.

We want

$$P(E \cap F') + P(E' \cap F) \overset{\text{indep.}}{=} P(E)P(F') + P(E')P(F)$$
$$= P(X_1 > \tfrac{1}{2})P(X_2 \le \tfrac{1}{2}) + P(X_1 \le \tfrac{1}{2})P(X_2 > \tfrac{1}{2}),$$
$$= 2P(X > \tfrac{1}{2})P(X \le \tfrac{1}{2}) \text{ where } X \sim f(x),$$

$$P(X > \tfrac{1}{2}) = \int_{1/2}^{1} (2-2x)\,dx = 2x - x^2\Big|_{1/2}^{1} = (2-1) - (1 - \tfrac{1}{4})$$
$$= 1 - \tfrac{3}{4} = \tfrac{1}{4};$$

and $P(X \le \frac{1}{2}) = 1 - P(X > \frac{1}{2}) = 1 - \frac{1}{4} = \frac{3}{4}$.

Thus the desired probability is

$$2\left(\tfrac{1}{4}\right)\left(\tfrac{3}{4}\right) = \frac{6}{16}.$$

3. A test for a disease correctly diagnoses a diseased person as having the disease with probability 0.85. The test incorrectly diagnoses someone without the disease as having the disease with a probability of 0.10. If 1% of the people in a population have the disease, what is the chance that a person from this population who tests positive for the disease actually has the disease?

A. .0085 B. .0791 C. .1075 D. .1500 E. .9000

Let D = event a person has the disease

T = event test diagnoses a person as having the disease.

Given $P(D) = 0.01$, or $P(D') = 0.99$

$P(T|D) = 0.85$, and $P(T|D') = 0.10$.

Want
$$P(D|T) = \frac{P(T|D)P(D)}{P(T|D)P(D) + P(T|D')P(D')}$$ by Bayes' Thm.
$$= \frac{(0.85)(0.01)}{(0.85)(0.01) + (0.10)(0.99)}$$
$$= \frac{0.0085}{0.0085 + 0.0990} = \frac{0.0085}{0.1075} = 0.0791$$

4. A box contains 10 marbles, of which 6 are red and 4 are green. A sample of size 3 is drawn without replacement from the box. What is the probability of at least 1 red marble, given that at least 1 of the marbles in the sample is green?

A. $\frac{5}{9}$ B. $\frac{9}{10}$ C. $\frac{24}{25}$ D. $\frac{4}{5}$ E. $\frac{8}{25}$

want
$$P(\text{at least } 1R \mid \text{at least } 1G) = \frac{P(\text{at least } 1R \text{ \& at least } 1G)}{P(\text{at least one } G)}$$

$$P(\text{at least } 1G) = 1 - P(\text{all red}) = 1 - \frac{\binom{6}{3}}{\binom{10}{3}} = \frac{\binom{10}{3} - \binom{6}{3}}{\binom{10}{3}}$$

$$P(\text{at least } 1R \text{ \& at least } 1G) = P(1R \text{ \& } 2G \text{ or } 2R \text{ \& } 1G)$$
$$= P(1R \text{ \& } 2G) + P(2R \text{ \& } 1G)$$
$$= \frac{\binom{6}{1}\binom{4}{2} + \binom{6}{2}\binom{4}{1}}{\binom{10}{3}}$$

$$\therefore P(\text{at least } 1R \text{ \& at least } 1G) = \frac{\binom{6}{1}\binom{4}{2} + \binom{6}{2}\binom{4}{1}}{\binom{10}{3} - \binom{6}{3}} = \frac{6 \cdot 6 + 15 \cdot 4}{120 - 20}$$
$$= \frac{96}{100} = \frac{24}{25}$$